Simulation of a Cosmic Ray Tomography Scanner for Trucks and Shipping Containers

A. Sh. Georgadze,[1,2] A. Giammanco,[3] V. A. Kudryavtsev,[4] M. Lagrange,[3] and C. Türkoğlu[4]

[1]*Institute of Physics, University of Tartu, W. Ostwaldi 1, 50411 Tartu, Estonia*

[2]*Institute for Nuclear Research, prospekt Nauky b.47, 03680 Kyiv, Ukraine*

[3]*Centre for Cosmology, Particle Physics and Phenomenology, Université catholique de Louvain, Chemin du Cyclotron 2, B-1348 Louvain-la-Neuve, Belgium*

[4]*Department of Physics and Astronomy, University of Sheffield, Hounsfield Road, Sheffield, S3 7RH, UK*

Corresponding author: A. Sh. Georgadze
Email address: a.sh.georgadze@gmail.com

Abstract

The SilentBorder project aims to develop and construct a new high-technology scanner for the identification of hazardous and illegal goods hidden in trucks and sea containers. The scanner will enable scanning of shipping containers or cargo and is based on muon tomography, a technology that uses natural cosmic ray muons and therefore is inherently safe for people. We report on the development of a simulation and reconstruction framework aimed at optimizing the geometry of the detector and exploring feasibility of CRT in real smuggling scenarios using simulated data. The framework includes GEANT4 modeling of light transport in a scintillating fiber tracker to optimize the geometry and materials used to produce fiber mats. A systematic comparison was made of particle generators such as CRY, MUSIBO, and EcoMug interfaced with the GEANT4 toolkit to find the most effective one for modeling real smuggling scenarios. The Point-of-Closest-Approach reconstruction algorithm was used to create 3D images of sea containers or trucks. An analysis of the sensitivity of CRT was performed using simulated synthetic data generated for different smuggling scenarios of contraband of low-Z organic materials and high-Z inorganic materials. Results of our research indicate that by using muon tomography, it would be possible to improve the performance and sensitivity of sea container and cargo screening systems to overcome limitations of traditional screening methods, such as X-ray scanners, when it comes to detecting illicit materials that may be well concealed. CRT can provide a complementary imaging technique that could enhance the detection capabilities of existing systems.

Keywords: radiation detector, applied physics, artificial intelligence, muon tomography
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1. INTRODUCTION

Muon tomography, or muography, is a technique that uses cosmic ray muons to produce images of large objects. It has been used for various applications such as underground applications [1, 2] and civil engineering [3], industrial applications [4], geoscience [5] and archaeology [6], nuclear waste characterization [7], nondestructive testing [8], noninvasive imaging of the interior of large structures, homeland security applications [9], and safeguards [10], for imaging shipping containers or cargo for contraband [11, 12]. When primary cosmic rays, predominantly protons and other atomic nuclei, collide with atoms in the Earth's atmosphere, they produce a shower of secondary particles, including muons. These muons are highly penetrating and can pass through various materials, including solid objects. The advantage of Muon tomography, also called cosmic ray tomography (CRT), for imaging shipping containers and trailers is its ability to provide a noninvasive and relatively rapid screening method. It allows for the detection of hidden compartments, anomalies, or contraband within the cargo without the need for physically opening or unloading the container. A recent review of the application of muon tomography to border security can be found in [13]. Two main techniques of CRT exist: absorption-based and scattering-based. The former takes advantage of the fact that the flux of muons reaching a particle detector is inversely related to the density of the material through which they pass. Dense materials, such as metals or high-density objects, attenuate or block more muons than less dense materials. Scattering-based tomography, first proposed in [14], exploits instead the scattering of muons off the nuclei of the materials traversed. The root-mean-square (RMS) deflection of the muons depends in fact on the atomic number (Z) of the material. This allows for elemental discrimination, thus making scattering-based muon tomography the technique of choice for cargo scanning [13]. In the rest of this paper, by CRT or muon tomography, we refer specifically to this technique. Compared to traditional X-ray scanners, CRTs provide more efficient imaging than X-rays through much thicker materials, such as thick layers of stone or metal. This makes CRT well suited for imaging applications in areas where traditional X-rays may face limitations. CRT was originally proposed to detect nuclear smuggling by scanning shipping containers for fissile materials. In the course of the SilentBorder project, the capabilities of muon scanning systems are extended to detect various kinds of threats and smuggling of goods in cargo containers and trucks.

2. MONTE CARLO SIMULATIONS

2.1. The SilentBorder System

SilentBorder is an H2020 research and innovation program for shipping container inspection at border control points. The goal of the SilentBorder project (`https://silentborder.eu/`) is to develop and validate a new high-technology CRT scanner for border guards, customs, and law enforcement authorities (LEAs), based on plastic scintillating fiber detectors. The SilentBorder system is composed of tracker modules, placed above, below, and on both sides of the container or truck to reconstruct the muon tracks passing the detectors. Each module consists of two detection planes, with a double layer of round (1 mm in diameter) plastic scintillating fiber arrays aligned orthogonal to each other glued on low-Z, low-density Rohacell sheets (see Figures 1(a) and 1(b)). The identification of the four struck fibers per detection plane yields X, Y, and Z coordinates of muon interaction points in space. Muons, when passing scintillating fibers, produce light detected by silicon photomultipliers (SiPM). Four muon interaction points in the top-bottom, top-lateral, lateral-bottom, and lateral-lateral modules facilitate the reconstruction of the initial and scattered muon trajectories. This, in turn, enables the creation of three-dimensional images of shipping container contents.

2.2. Monte Carlo Simulations of the Detector

To optimize the performance of the CRT system, dedicated studies were performed using a detailed simulation of the system using GEANT4 toolkit [15]. The detailed schemes of detector modules allow us to perform simulation of optical photon transport to evaluate optimal fiber length for best light collection as well as acquiring muon momentum and energy deposited in fiber by passing muons which allow the development of track reconstruction and detector alignment algorithms. The Monte Carlo simulations have allowed us to select the best placement for the scintillating fiber detector aimed to improve the detection efficiency of the scanner and optimize distance between tracking detectors to obtain the best track reconstruction accuracy. The detailed scheme of XY-plane consisting of orthogonal double-layer scintillating fiber ribbons has been constructed in Geant4 for the development of muon track reconstruction and detector alignment algorithms (in Figure 1(c), fibers hit with muons are visualized in cyan color). At the same time, simulations have been useful to define the best size of fiber ribbons and the hole detector module.

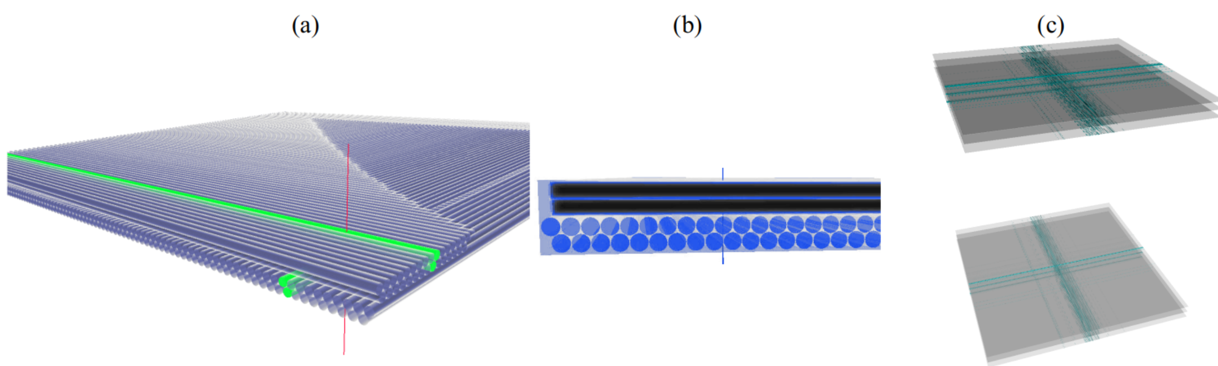

FIGURE 1: Layout of the plastic scintillating fiber lattice in GEANT4 (a); the scheme of position-sensitive planes (providing X- and Y-coordinates) constructed in Geant4, consisting of orthogonal double-layer scintillating fiber detector planes (b); a multilayer detector scheme constructed in Geant4 for the development of muon track reconstruction and detector alignment algorithms (c).

2.3. Muon Event Generator

We have performed a study to compare parametric particle generators such as Cosmic-ray shower generator (CRY) [16], EcoMug [17], and MUSIBO [18] interfaced with the GEANT4 toolkit. CRY is a software library based on precomputed input tables obtained from full MCNPX simulations of primary cosmic rays in the atmosphere. It generates correlated distributions of cosmic ray fluxes at one of three altitudes (sea level, 2100 m, and 11300 m), which are used as input to the Geant4 detector simulation code. In addition to muons, CRY can simulate neutrons, protons, electrons, photons, and pions. The generator takes into account solar modulation, latitude-dependent geomagnetic cutoff, and altitude location dependence. Simulated particles are sampled on a flat surface. The EcoMug is a parametric cosmic muon generator, where the flux of muons at Earth's surface is modeled from the experimental data. The user can choose cosmic ray muon sampling on flat, cylindrical, and hemispherical surfaces. MUSIBO is a parametric generator, where the flux and kinematics of muons are based on Gaisser's parameterization [19] of the muon spectrum and angular distribution, modified to account for muon decay and curvature of the Earth's surface. MUSIBO generates muons on the surfaces of a box (rectangular parallelepiped) and flat surfaces. All generators demonstrate similar performance in the reconstruction of cargo container content. We decided to use the CRY software as the main generator because of its ability to take into

TABLE 1: Summary of the results from simulations with the CRY, MUSIBO, and EcoMug generators.

	Muon generators				
Tracker pairs	CRY (plane)	MUSIBO (box)	MUSIBO (plane)	EcoMug (plane)	EcoMug (hemisphere)
top-bottom	1.10×10^6	1.25×10^6	1.20×10^6	1.05×10^6	1.20×10^6
top-lateral	0.86×10^6	1.08×10^6	0.94×10^6	0.86×10^6	0.92×10^6
lateral-bottom	0.69×10^6	1.07×10^6	0.68×10^6	0.66×10^6	0.89×10^6
lateral-lateral	5.70×10^4	1.27×10^5	4.80×10^4	5.80×10^4	1.03×10^5

account latitude and altitude location dependencies, and MUSIBO is introduced as a tool to control systematic uncertainties associated with inaccuracies in approximating the muon flux by the generators. Additionally, we simulated the passage of 10 million muons through the CRT system (11 m $\times$ 11 m for plane surfaces for CRY, MUSIBO, and EcoMug Sky generators, 11 m in diameter for the EcoMug hemisphere generator, and 11 m $\times$ 5 m $\times$ 4.5 m for the MUSIBO BOX generator) and counted reconstructed muon tracks that passed through the top-bottom, top-lateral, lateral-bottom, and lateral-lateral trackers. In Table 1, we present a summary of the results from simulating muon passage in the CRT system using the CRY, MUSIBO, and EcoMug generators.

3. PERFORMANCE EVALUATION OF THE SilentBorder SCANNER

In order to evaluate the performance of the CRT scanner, Monte Carlo simulations have been performed with different scenarios of loaded cargo. We considered carton boxes placed on pallets that are filled with various organic goods, including clothes, fruits, and dry pasta. Additionally, these scenarios encompass substances like sand, cement, soil, or steel products such as steel beams, rebars, sheets, and various industrial products and electronics. The simulated 20-foot ISO container is a standard-sized rectangular box ($609.6 \times 259.1 \times 243.8\,\mathrm{cm}^3$) used to transport goods by ship, train, or truck. The payload, which is the maximum loading capacity, is 25000 kg for a 20-foot container. The capacity is 33 cubic meters. As a source of data for the simulation of cargo contents and average densities of cargo shipments, one can use a PIERS (Port Import/Export Reporting Service) United States import data set. It can be used to understand containerized cargo traffic and cargo material composition. This allow the creation of loading scenarios formalized in custom declarations. According to research [20], the mean cargo density is just under $0.2\,\mathrm{g/cm}^3$. Contraband materials can be hidden among the legal materials or simply randomly replaced in packaging legal materials.

For testing Geant4 models of shipping containers and cargo materials, we use simplified detector design—rectangular planes as ideal detectors (Figure 2(a)). We simulated 10 million muons sampled on the surface 10 m $\times$ 10 m using a CRY generator, which corresponds to approximately 10 minutes of measurement. The measurement time is highly variable as it depends on the materials being scanned. In most cases, when scanning organics with a density of 0.1–0.2 $\mathrm{g/cm}^3$, 2–5 minutes are typically sufficient. However, for materials with larger density (more than $1.0\,\mathrm{g/cm}^3$), a scanning time of 10 minutes or more is required.

A simple Point-of-Closest-Approach (PoCA) method was used [21] for the reconstruction of 3D images from simulated data. This method makes the simplified assumption that the muon scattering occurs in a single point and allows for a quick test of developed Geant4 geometries of real scenarios. The result of reconstruction using the PoCA algorithm returns a set of PoCA points and scattering angles at each point. Tomographic reconstructions and analyses were performed by using the ROOT package [22]. An example of tomographic reconstruction of an empty container, in particular XY and YZ projections of a tomography image, is shown in Figures 2(b) and 2(c).

3.1. Monte Carlo Simulation of the Scenarios of Cargo Loading with Legal Goods and Smuggling

In addition to simulating the detector, another task involves creating scenes in Geant4 or importing them from CAD software. These scenes are built using realistic custom configurations to mimic actual smuggling scenarios. The realistic scenarios include the normal loading of containers with legal goods and scenarios with illegal or threatening materials hidden inside normal goods. There are several tips for better shipping container loading. A master case is a bag with bulk materials or a carton box that contains multiple units and sometimes also includes inner packs, which constitute another level of packaging. To facilitate handling and storage, cargo is usually placed on pallets. A pallet is a shipping platform on which multiple master cases are shipped. Cargo is packed into master cases, which, in turn, are packed into pallets for shipment. Figure 3(a) shows a GEANT4 visualization of cardboard boxes of clothing measuring $50 \times 40 \times 25\,\mathrm{cm}^3$ (width $\times$ depth $\times$ height) arranged on pallets. A standard 20-foot ISO container can hold 11 Euro pallets or 10 standard pallets. The 2D projection of the simulated tomographic image shows a uniform scattering density distribution, allowing even pallets loaded with clothing to be resolved (Figures 3(b) and 3(c)). Low-Z materials can be identified in these images and compared with those expected from the custom manifest. The constructed GEANT4 models of smuggling scenarios are used to generate synthetic data. This data is then used to train Machine Learning and Artificial Intelligence algorithms, allowing us to distinguish between 3D images of illegal and legal cargo. These models also help in developing procedures and algorithms for the automated identification of potentially illegal cargo.

In Figure 4(a), on each pallet, one randomly selected box of clothes (density: $0.2\,\mathrm{g/cm}^3$) is replaced with a box containing an explosive with a density of $1.812\,\mathrm{g/cm}^3$. Figures 4(b) and 4(c) show a 2D projection of the tomographic image for the scenario of explosives among custom goods. In these images, among legal low-Z materials, boxes with explosive materials can be visualized. To obtain a clearer image of illegal goods and a more accurate estimate of their density, the reconstructed image can be segmented

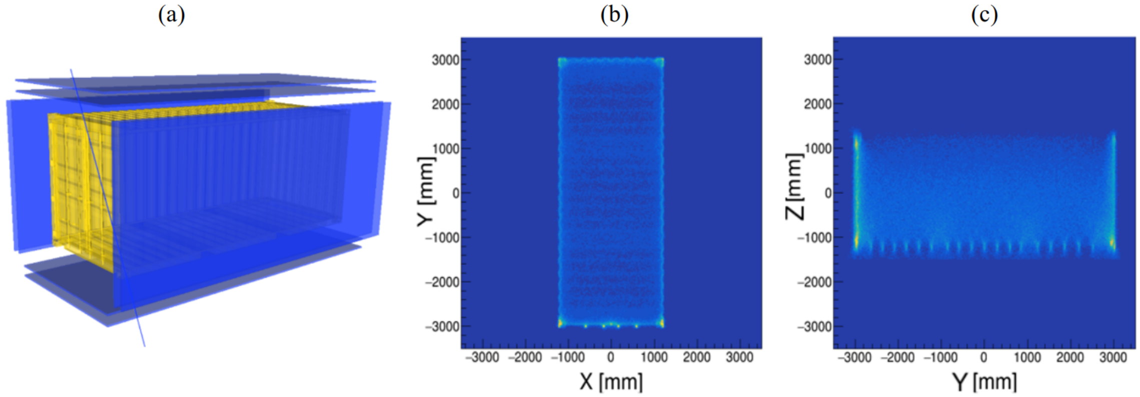

FIGURE 2: Visualization of CRT system with shipping container using GEANT4 (a), XY (b) and YZ (c) projections of a tomography image of empty shipping container.

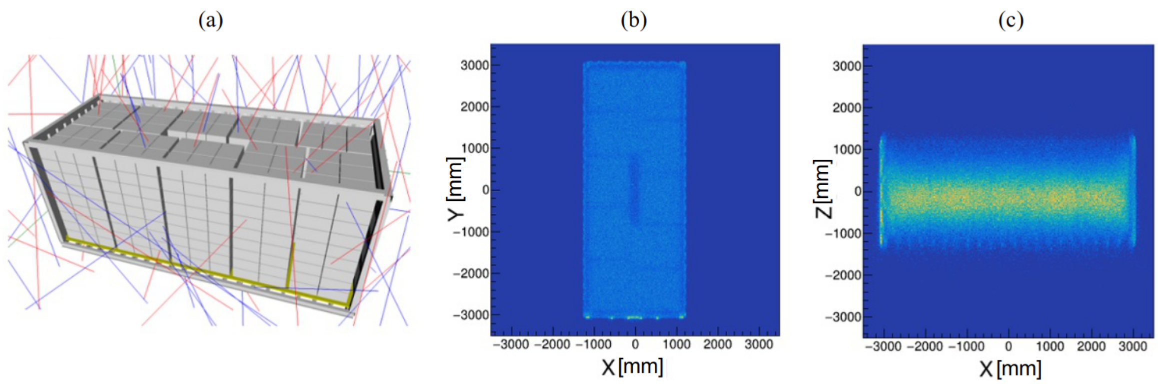

FIGURE 3: GEANT4 visualization of shipping container loaded with cloths in carton boxes placed on pallets (a). XY (b) and YZ (c) are 2D projections of a tomographic image of the container with clothes.

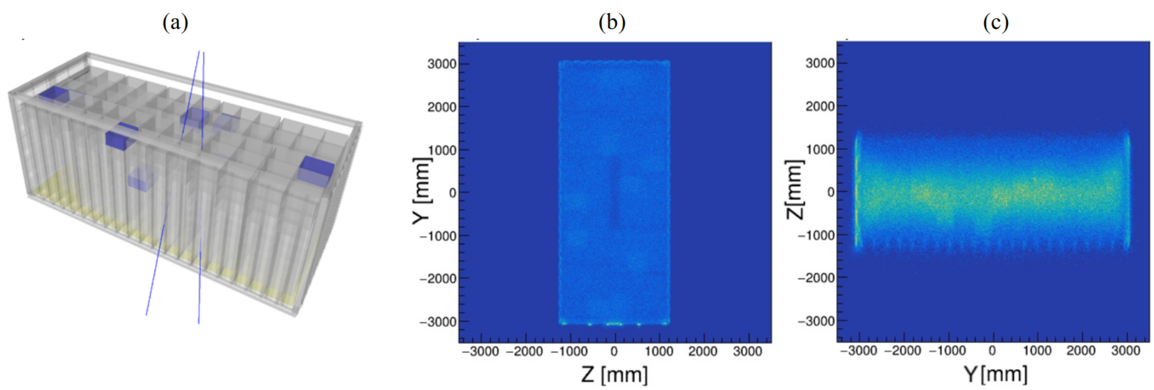

FIGURE 4: GEANT4 visualization of the shipping container in which one bag in each pallet is loaded with explosives (bags in blue color) (a); XY (b) and YZ (c) projections of a tomographic image of shipping container in which one randomly selected bag on each pallet is loaded with explosives.

into slices to improve the signal-to-background ratio. We select slice height corresponding to a layer of boxes measuring 25 cm. On the resulting 2D projections of slices (see Figure 5), boxes loaded with explosives became much more detectable.

Another scenario we considered involves the smuggling of tobacco products hidden within wood pellets, wood bricks, or wooden boards. The density of wood varies from about 0.3 to 1.6 g/cm^3. A GEANT4 model of the semitrailer truck ($13.6 \times 2.7 \times 2.45\,m^3$) was created (Figure 6). In this case, contraband cigarettes (density 0.18 g/cm^3) are hidden in a stack of wooden boards (density 0.8 g/cm^3). The wood planks are arranged in two stacks in height and width, and six in length. We simulated 29 million muons using the CRY muon generator, sampling muons on a surface area of 20 m $\times$ 20 m, which is equivalent to approximately 650 seconds of measurement. Additionally, for comparison, we simulated 20 million muons using the MUSIBO generator, with

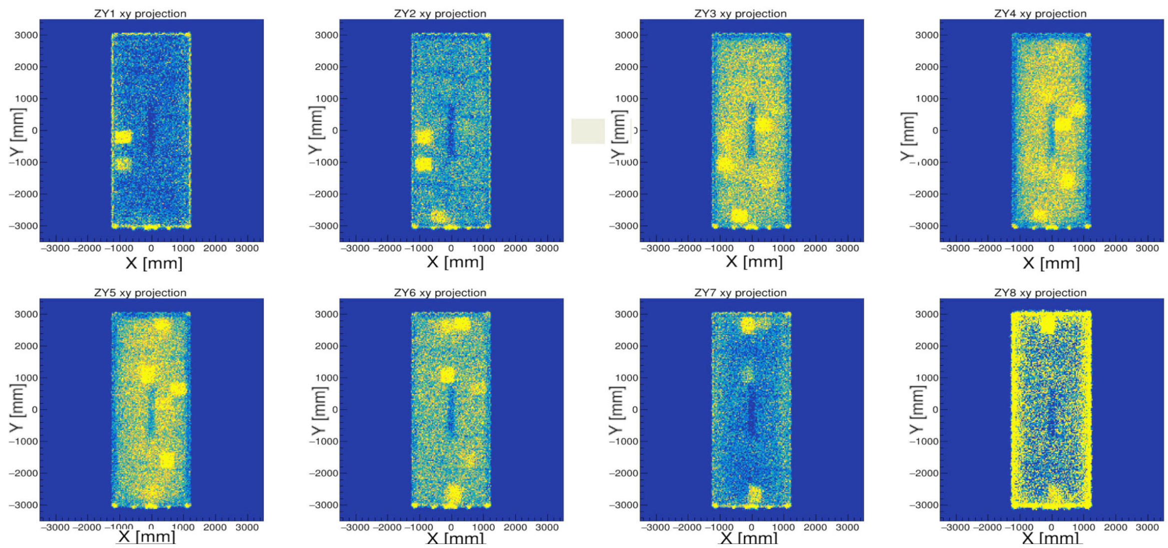

FIGURE 5: 2D projections of 3D slices of a tomographic image of a shipping container in which one bag in each pallet is loaded with explosives.

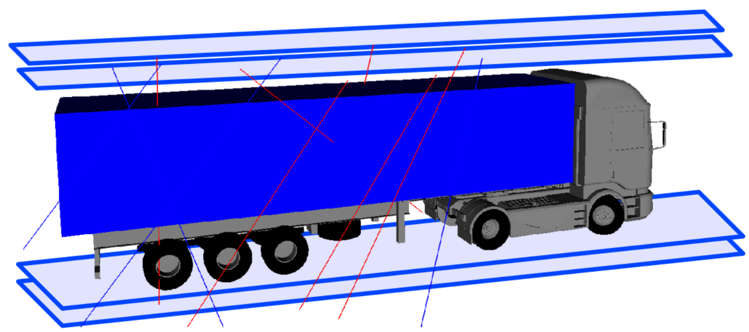

FIGURE 6: Visualization of Geant4 simulation of a truck with semitrailer scanning using muons.

muons sampled on the surface of a box measuring 21.0 m $\times$ 4.5 m (830 seconds of measurement). The shapes of the images are identical, but there are somewhat more reconstructed PoCA points in the case of using the MUSIBO generator. We performed the same processing on the truck images as we did for the container analysis. The simulated results are presented in Figure 7. One can see that blocks of wood boards replaced with cigarettes can be easily identified.

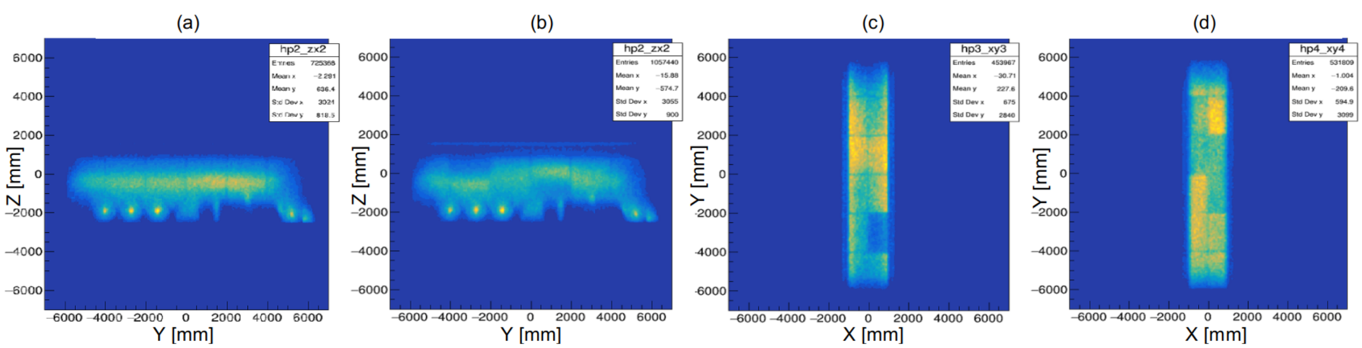

FIGURE 7: 2D projections of tomography image of loading truck with legal cargo (a), 2D projections of tomography image of loading truck with contraband of tobacco (b), projections of 100 cm high tomographic image slices with the contraband tobacco hidden among stacks of wooden planks loaded on a semitrailer.

This project is also a testing ground for all developments of TomOpt [23], an ML-based software aiming at the holistic optimization of the main muon scanner parameters. It includes modules for parametric simulation and reconstruction of the muon interactions, validated versus Geant4-based simulations. TomOpt is developed to optimize the geometrical layout and specifica-

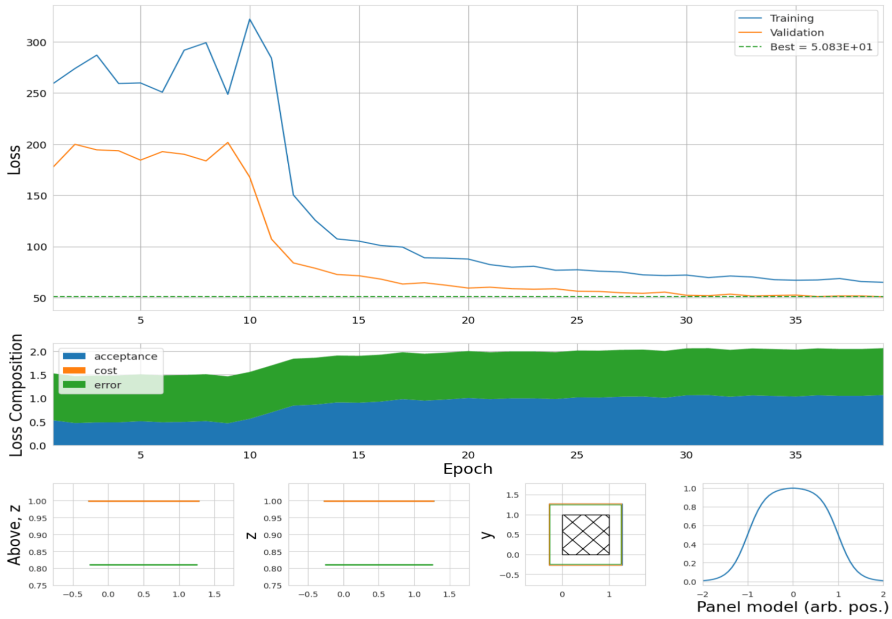

FIGURE 8: An example of a TomOpt optimization cycle for detector parameters optimization.

tions of detectors designed for CRT. The software includes a parametric modeling of muon interactions with detectors and scanned volumes, the inference of radiation length in each voxel, and based on the novel paradigm of differentiable programming [24], it includes an optimization cycle performing the minimization of a suitable figure of merit which encompasses terms that depend on the accuracy of the estimation, on the amount of time needed to obtain such estimation, and on the cost of the CRT apparatus. Figure 8 illustrates the optimization cycle during the training of the algorithm. The optimization of the detector design and performance is currently studied using TomOpt software. We are also continuing the development of Geant4 models to simulate a wide variety of common container loading and smuggling scenarios. This effort is aimed at the development of artificial intelligence tools for automatic smuggling detection.

In muon tomography, the scattering angle distribution of muons after passing through different materials is one way to characterize these materials. This distribution is calculated using the incoming and the outgoing tracks of muons simulated using GEANT4. In order to study the effect on muon scattering angle for different materials, we simulate muons with a mean energy of 7.16 GeV. The muons pass through a volume of interest with dimensions 1100 cm $\times$ 500 cm $\times$ 450 cm consisting of various low- and high-Z materials such as nicotine, muscle, trinitrotoluene (TNT), hexogen (RDX), and lead. The scattering angle of muons depends largely on the atomic number (Z) of materials they are passing through. Figures 9(a) and 9(b) show the simulated scattering angle distribution for various materials of 10 cm (a) and 50 cm (b) thicknesses for 10 million initial muon events. These plots show that for low-Z materials, simulated scattering angle distributions are very close and more difficult to resolve. Figures 9(c) and 9(d) show scattering angle distributions of muons for nicotine (c) and lead (d) of 20 cm thickness each plotted against the initial muon energies. The scattering angle-energy distribution of lead is more widespread due to having a much higher atomic number compared to the nicotine molecule.

4. CONCLUSIONS

A complete Monte Carlo simulation chain was developed for the SilentBorder project to assess the performance of CRT scanners. A study was conducted comparing three different cosmic ray muon generators—CRY, EcoMug, and MUSIBO—which concluded that all generators demonstrated compatible performance in imaging ISO container content. Geant4 models were developed to simulate container and semitrailer trucks, focusing on a range of smuggling scenarios involving shipping containers and trucks. These simulations successfully demonstrated the capability to detect low-Z smuggled materials within low-Z custom goods. In the future works, a large variety of smuggling scenarios will be simulated to perform training artificial intelligence algorithms for automated detection of contraband and illegal materials.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

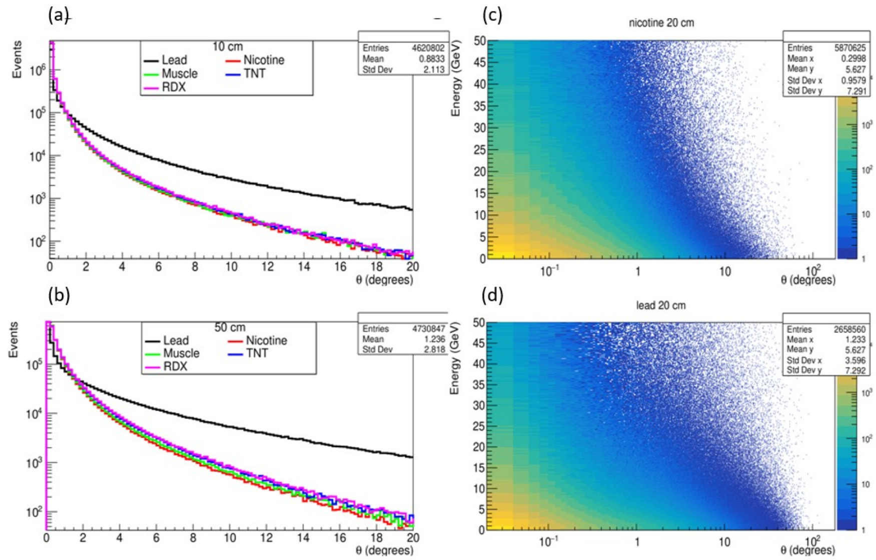

FIGURE 9: Scattering angle distributions for cosmic ray muons calculated by the incoming and outgoing muon tracks traversing various materials with thicknesses 10 cm (a) and 50 cm (b) with different Z. Scattering angle distributions for nicotine (c) and lead (d) of 20 cm thicknesses plotted with respect to the initial muon energies.

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